

As always, show your work and follow the HW format. You may use Excel, but must show sample calculations.

1. Single Mean. A new roof truss is designed to hold more than 5000 pounds of snow load. You test 20 trusses and obtain a mean of 5025 pounds and standard deviation of 80 pounds. Use the 8-step method to determine if the mean is greater than 5000 pounds, at a significance of 0.05.

**SOLUTION:**

1.  $\mu$
2.  $H_0: \mu = 5000$  lb
3.  $H_1: \mu > 5000$  lb
4.  $\alpha = 0.05$
5.  $TS = T$  (because  $\sigma$  unknown and  $n < 30$ )
6. Reject  $H_0$  if  $TS > CV$ , where  $CV = t_{\alpha, v} = t_{0.05, 19} = 1.73$  [Table A.2 or T.INV(0.95, 19)]
7. Calculate necessary values:
  - a.  $T_o = \frac{\bar{X} - \mu_o}{s/\sqrt{n}} = \frac{5025 - 5000}{80/\sqrt{20}} = 1.40$
8.  $1.40 < 1.71$ . Fail to Reject  $H_0$  at  $\alpha = 0.05$ ; the capacity of the roof truss appears to NOT be greater than 5000 lb.

2. Two Means. Two different types of tubing should have different maximum pressures. Use the sample data given below to assess this using the 8-step method, using a significance of 0.05.

Sample Parameter	Tube 1	Tube 2
Size	22	25
Mean, psi	150	160
Standard Deviation, psi	23	15

**SOLUTION:**

- $\mu_1 - \mu_2$
- $H_0: \mu_1 - \mu_2 = 0$
- $H_1: \mu_1 - \mu_2 \neq 0$
- $\alpha = 0.05$
- TS = T\* (because  $\sigma$ 's unknown and unequal, and both  $n < 30$ )
- Reject  $H_0$  if  $|TS| > CV$ , where  $CV = t_{\alpha/2, v} = t_{0.025, 36} = 2.03$  [Table A.2 or T.INV(0.95,36)]
  - $$v = \frac{\frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{\frac{(\frac{s_1^2}{n_1})^2}{n_1+1} + \frac{(\frac{s_2^2}{n_2})^2}{n_2+1}} - 2 = \frac{(\frac{23^2}{22} + \frac{15^2}{25})^2}{\frac{(23)^2}{22+1} + \frac{(15)^2}{25+1}} - 2 = 36$$
- Calculate necessary values:
  - $$T_o^* = \frac{(150-160)-0}{(\frac{23^2}{22} + \frac{15^2}{25})^{0.5}} = -1.74$$
- $|-1.74| < 2.03$ . Fail to Reject  $H_0$  at  $\alpha = 0.05$ ; The pressure capacities are not significantly different.

3. Distribution Checking. Can parking spot pavement defects be predicted by the Poisson Distribution? You evaluate almost 1800 parking spots and observe the following.

Defects in parking spot, $d_i$	Parking spots with $d_i$ defects, $n_i$
0	720
1	650
2	330
3	90
$\geq 4$	5

Use the 8-step method with a significance level of 0.1. “r” will be the average number of defects per parking spot. Determine r as the (total number of defects observed) / (total number of parking spots evaluated).

**Solution:**

Distribution Checking Table

Count	$n_i$	$p_i$ 736.122	$e_i$	$c_i$
0	720	0.41010	736.122	0.4
1	650	0.36554	656.153	0.1
2	330	0.16292	292.436	4.8
3	90	0.04841	86.889	0.1
$\geq 4$	5	0.01304	23.400	14.5
<b>Sums →</b>	<b>1795</b>	<b>1</b>	<b>1795</b>	<b>19.8</b>

Sample Calculations:

$$r = (0 \cdot 720 + 1 \cdot 650 + 2 \cdot 330 + 3 \cdot 90 + 4 \cdot 5) / 1795 = 0.8914$$

$$\Pr(X = x) = \frac{r^x}{x!} e^{-r}, \text{ e.g., for } X = 0: \frac{0.8914^0}{0!} e^{-0.8914} = 0.4101 \text{ and } \Pr(X \geq 4) = 1 - \text{sum of 0 to 3 counts.}$$

$$e_i = p_i \cdot 1795, \text{ e.g., for the first row, } e_1 = 0.4101 \cdot 1795 = 736.122.$$

$$c_i = (n_i - e_i)^2 / e_i, \text{ e.g., for the first row, } c_1 = (720 - 736.122)^2 / 736.122 = 0.40.$$

$$C_0 \text{ equals the sum of the } c_i \text{ column} = 19.8.$$

## CE Systems – Hypothesis Testing

1.  $C$  (Stand in for all  $n_i = e_i$ )
2.  $H_0: C = 0$  (All  $n_i = e_i$ )
3.  $H_1: C > 0$  (At least one  $n_i \neq e_i$ )
4.  $\alpha = 0.1$
5.  $TS = C$
6. Reject  $H_0$  if  $TS > CV$ , where  $v = 5 - 1 - 1 = 3$ , and  $CV = \chi^2_{0.1,3} = 6.25$  [Table A.3 or CHISQ.INV.RT(0.1,3)]
7. Calculate necessary values: see Table,  $c_o = 19.8$
8. Reject or Fail to Reject based on rejection equation:  $19.8 > 6.25$ ; Reject  $H_0$  at significance level 0.1, the Poisson distribution does not appear to model the parking spot pavement defects.

4. Simple Linear Regression. Given the data below, use the 8-step method to determine if the overall linear relationship is significant at the 0.01 significance level. Use a “TS > CV” rejection region. You will need to calculate  $f_o$  and  $f_{\alpha,u,v}$ .

X	Y	$s_{xy}$	$s_{xx}$	$\hat{y}_i$	$SS_R$	$SS_T$	$SS_E$
4	5	84.00	110.25	6.42	43.28	64.00	2.02
6	9	34.00	72.25	7.67	28.37	16.00	1.76
10	8	22.50	20.25	10.18	7.95	25.00	4.75
14	15	-1.00	0.25	12.69	0.10	4.00	5.35
23	20	59.50	72.25	18.33	28.37	49.00	2.80
30	21	124.00	240.25	22.71	94.32	64.00	2.93
$\bar{X} \downarrow$	$\bar{Y} \downarrow$	$s_{xy} \downarrow$	$s_{xx} \downarrow$	NA	$SS_R \downarrow$	$SS_T \downarrow$	$SS_E \downarrow$
<b>14.50</b>	<b>13.00</b>	<b>323.00</b>	<b>515.50</b>	<b>NA</b>	<b>202.38</b>	<b>222.00</b>	<b>19.62</b>

**SOLUTION:**

1. F (Stand in for slope coefficients,  $\beta_j$ )
2.  $H_0: F = 0$  (All  $\beta_j = 0$ )
3.  $H_1: F > 0$  (At least one  $\beta_j \neq 0$ )
4.  $\alpha = 0.1$
5.  $TS = F$
6. Reject  $H_0$  if  $TS > CV$ ,  $u = p - 1 = 2 - 1$  and  $v = n - p = 5 - 2$ ,  $CV = f_{0.01,1,4} = 21.2$  [Table A.5 or F.INV.RT(0.01,1,4) or F.INV(0.9,1,3)]
7. Calculate necessary values:  
 $TS = f_o = MS_R/MS_E = (SS_R/(p-1))/(SS_E/(n-p)) = (202.38/1)/(19.62/4) = 41.3$
8. Reject or Fail to Reject based on rejection equation:  $41.3 > 21.2$ ; Reject  $H_0$  at significance level 0.01, a significant linear relationship exists between the dependent variable and the independent variable.

5. Multiple Linear Regression. Use A and B to predict Y. Use the Excel Data Analysis Add-in.

A	B	Y
22	1	8
17	3	10
10	4	11
14	6	14
23	10	21

Include the Excel Data Analysis Add-in output and use it to answer the following questions.

(a) Use the 8-step method to determine if the overall linear relationship is significant at the 0.01 significance level. Use a “p-value <  $\alpha$ ” rejection region.

(b) Use the 8-step method to determine if  $\beta_1$  is not equal to zero (at the 0.01 significance level). Use a “p-value <  $\alpha$ ” rejection region.

(c) Use the 8-step method to determine if  $\beta_2$  is not equal to zero (at the 0.01 significance level). Use a “p-value <  $\alpha$ ” rejection region.

**SOLUTION:**

ANOVA					
	df	SS	MS	F	Significance F
Regression	2	102.6926505	51.34632526	956.6198887	0.001044256
Residual	2	0.107349483	0.053674742		
Total	4	102.8			

	Coefficients	Standard Error	t Stat	P-value
Intercept	3.911314	0.391610751	9.987758232	0.009876267
X Variable 1	0.116382	0.021652021	5.375127114	0.032912464
X Variable 2	1.434773	0.034497192	41.59100506	0.000577597

(a)

1. F (Stand in for slope coefficients,  $\beta_j$ )
2.  $H_0: F = 0$  (All  $\beta_j = 0$ )
3.  $H_1: F > 0$  (At least one  $\beta_j \neq 0$ )
4.  $\alpha = 0.01$
5.  $TS = F$
6. Reject  $H_0$  if P-Value <  $\alpha$ , from Table P-Value = 0.00104
7. Calculate necessary values: NA
8. Reject or Fail to Reject based on rejection equation:  $0.00104 < 0.01$ ; Reject  $H_0$  at significance level 0.01, a significant linear relationship exists between the dependent variable and the independent variables.

(b)

1.  $\beta_1$
2.  $H_0: \beta_1 = 0$
3.  $H_0: \beta_1 \neq 0$
4.  $\alpha = 0.01$
5. T
6. Reject  $H_0$  if  $p\text{-value} < \alpha$ , where  $p\text{-value}$  is determined from the Excel Data Analysis Add-in Table given above.  $p\text{-value} = 0.0329$  (From Table above)
7. Calculate necessary values: NA
8. Reject or Fail to Reject  $H_0$ :  $0.0329 > 0.01$ ; Fail to Reject  $H_0$  at  $\alpha = 0.05$ ,  $\beta_1$  is not significantly different from zero.

(c)

1.  $\beta_2$
2.  $H_0: \beta_2 = 0$
3.  $H_0: \beta_2 \neq 0$
4.  $\alpha = 0.01$
5. T
6. Reject  $H_0$  if  $p\text{-value} < \alpha$  where  $p\text{-value}$  is determined from the Excel Data Analysis Add-in Table given above.  $p\text{-value} = 0.000578$  (From Table above)
7. Calculate necessary values: NA
8. Reject or Fail to Reject  $H_0$ :  $0.000578 < 0.05$ ; Reject  $H_0$  at  $\alpha = 0.01$ ,  $\beta_2$  is significantly different from zero.